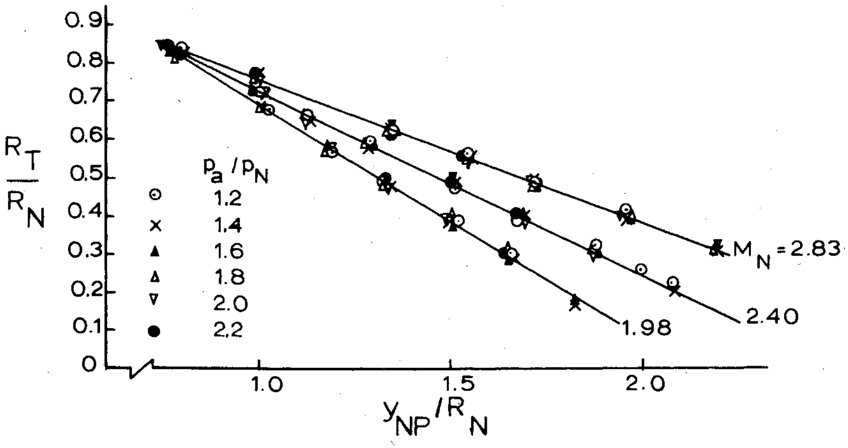

Fig. 2 Measured variation of triple point radius with plate position.

centered expansion fan which occurs at the edge of the jet shock. It is known that this centered expansion fan is the principal factor in determining the structure of the near wall jet.[9]

A Further Experimental Observation

As well as shock heights, the radial location, R_T, of the triple point was measured from the schlieren pictures. It was found that the value of R_T varied linearly with y_{NP} but was curiously insensitive to the overexpansion ratio, p_a/p_N. Figure 2 shows the results obtained with three different nozzle exit Mach numbers. These results do not mean that the shock structure is invariant with p_a/p_N. Indeed the jet shock angle increases as p_a/p_N increases and Δ_T must also increase in order to preserve a constant value of R_T. Even more surprisingly, in each case, the line through the experimental points of Fig. 2 makes an angle with the y_{NP} axis which equals the Mach angle, μ_N, in the exit plane. It can also be seen that each of the lines pass through the point $y_{NP} = 0.75$, $R_T = 0.845 R_N$, thus enabling the entire set of results for R_T to be represented by the expression

$$R_T = (0.845 + 0.75 \tan\mu_N) R_N - y_{NP} \tan\mu_N$$

From this, an expression for the triple point height Δ_T can be obtained. The jet shock angle changes slightly as it is propagated downstream but can be well represented by the arithmetic mean of its values at the nozzle lip (β_N) and at the triple point (β_T). Elementary geometry and the above equation then lead to the expression

$$\Delta_T = y_{NP}(1 - \tan\mu_N \cot\bar{\beta}) + R_N(0.75\tan\mu_N - 0.155)\cot\bar{\beta}$$

where $\bar{\beta} \equiv 0.5(\beta_N + \beta_T)$. In many cases, it will be sufficient to take $\bar{\beta} = \beta_N$.

No physical explanation is offered for this curious collapsing of the data. Indeed, it may have no physical significance. However, it is a little easier to see what is happening if one observes that, if Δ_T were independent of y_{NP}, then the lines for R_T would make an angle with the y_{NP} axis equal to the shock angle: the fact that the angle μ_N occurs instead is due to the increase of Δ_T with y_{NP} which can be seen in Fig. 1.

Conclusions

The BGS method is not satisfactory for impinging overexpanded jets. The most likely reason for this being the neglect of the tail shock flow. Simple empirical expressions have been obtained for radial and axial locations of the triple point formed by the intersection of the plate shock with the jet shock.

References

[1]Gummer, J.H. and Hunt, B.L. "The Impingement of a Uniform Axisymmetric, Supersonic Jet on a Perpendicular Flat Plate," *The Aeronautical Quarterly*, Vol. 22, Nov. 1971, pp. 403-420.

[2]Belotserkovskii, O.M. "Flow with a Detached Shock Wave about a Symmetrical Profile," *Journal of Applied Mathematics and Mecanics (PMM)*, Vol. 22, 1958, pp. 279-296.

[3]Gummer, J.H. and Hunt, B.L. "The Impingement of Non-Uniform, Axisymmetric, Supersonic Jets on a Perpendicular Flat Plate," *Israel Journal of Technology*, Vol. 12, 1974, pp. 221-235.

[4]Belov, I.A., Ginzburg, I.P., and Shub, L.I., "Supersonic Under-Expanded Jet Impingement upon a Flat Plate," *International Journal of Heat and Mass Transfer*, Vol. 16, Nov. 1973, pp. 2067-2076.

[5]Henderson, L.F. "The Three-Shock Confluence on a Simple Wedge Intake," *The Aeronautical Quarterly*, Vol. 16, Feb. 1975, pp. 42-54.

[6]Kalghatgi, G.T. and Hunt, B.L. "The Three-Shock Confluence Problem for Normally Impinging, Over-Expanded Jets," *The Aeronautical Quarterly*, Vol. 26, May 1975, pp. 117-132.

[7]Kalghatgi, G. T., "Some Aspects of Supersonic Jet Impingement on Plane Perpendicular Surfaces," Ph.D. Thesis, 1975, University of Bristol, England.

[8]Kalghatgi, G.T. and Hunt, B.L. "The Occurrence of Stagnation Bubbles in Supersonic Jet Impingement Flows," *The Aeronautical Quarterly*, Vol. 27, Aug. 1976, pp. 169-185.

[9]Carling, J.C. and Hunt, B.L. "The Near Wall Jet of a Normally Impinging, Uniform, Axisymmetric, Supersonic Jet," *Journal of Fluid Mechanics*, Vol. 66, pt. 1, Oct. 1974, pp. 159-176.

Attitude Control of Spinning Spacecraft by Radiation Pressure

K. C. Pande*

Indian Institute of Technology, Kanpur, India

Nomenclature

A_i, ϵ_i	= control plate area and moment arm, respectively, $i = 1,2$
I_s, I_t	= moments of inertia of the satellite about the symmetry and transverse axes, respectively
O	= center of the Earth
S	= center of mass of the satellite
$\bar{i}, \bar{j}, \bar{k}$	= unit vectors along x, y, and z axes, respectively
$\bar{n}_i$	= unit vector along plate normal, $i = 1,2$
p	= solar radiation pressure, 4.65×10^{-6} N/m^2
$\bar{u}$	= unit vector in the direction of the sun, $u_x\bar{i} + u_y\bar{j} + u_z\bar{k}$
u_x	$= \cos\sigma\cos\phi + \sin\sigma\cos i\sin\phi$
u_y	$= -(\cos\sigma\sin\phi - \sin\sigma\cos i\cos\phi)\cos\theta - \sin\sigma\sin i\sin\theta$
u_z	$= (\cos\sigma\sin\phi - \sin\sigma\cos i\cos\phi)\sin\theta - \sin\sigma\sin i\cos\theta$
ρ	= reflectivity of control surface
$\bar{\omega}$	= angular velocity of satellite, $\omega_x\bar{i} + \omega_y\bar{j} + \omega_z\bar{k}$

Received Feb. 13; revision received June 3, 1976.

Index category: Spacecraft Attitude Dynamics and Control.

*Lecturer, Department of Mechanical Engineering.

Introduction

THE concept of using solar radiation pressure for attitude control of spinning spacecraft is well recognized. Several authors[1-4] have considered passive orientation of the spin-axis along the sun-satellite line employing such devices as a body-fixed corner mirror array, spring-mounted solar paddles, etc. Libration damping and attitude control in an orbiting reference frame were studied by Modi and Pande[5] for the case of satellites with very low spin rates. The authors suggested a semipassive solar controller consisting of several sets of rotatable control surfaces. Pande et al.[6] applied a similar approach to the time-optimal pitch control of an unsymmetrical satellite.

The general problem of imparting any inertially fixed orientation to the spacecraft spin-axis by means of solar radiation pressure, however, remains virtually unexplored. On the other hand, such a capability would represent a relatively inexpensive method of satellite acquisition in the desired attitude subsequent to orbital injection. Furthermore, it would also enable a satellite to undertake a multiple mission wherein several different space orientations may be desired over a period of time.

The present objective is to develop a solar pressure control system capable of providing any inertially fixed attitude to the spacecraft spin-axis. A controller configuration employing two mirror-like surfaces is considered and a general control law leading to asymptotic stability of the system is derived. The performance capability of the controller is examined in conjunction with a simple but practical control strategy.

Formulation of the Problem

The geometry of the orbital and attitude motion of an axisymmetric satellite is shown in Fig. 1. With reference to the inertial frame x_0, y_0, z_0, the spatial orientation of the axis of symmetry is completely specified by two successive Eulerian rotations ϕ and θ, referred to as the azimuth and the elevation angle, respectively. The satellite spins in the principal axis frame x,y,z with angular velocity $\dot\psi$.

The proposed solar controller consists of two light, highly reflective plates P_1 and P_2 which may be rotated about the axis of symmetry of the satellite. The rotations β_i $(i=1,2)$, measured from the x-axis, may be suitably maneuvered to vary the solar radiation torque controlling the spin-axis orientation. In order to avoid any mutual cancellation of the moments due to the two control plates, only one of them operates at a given time. The nonoperative plate may be turned "off" by maintaining it parallel to the sun-line. The control U identifies the active and the nonactive plates. $U=+1$ implies that the control surface P_1 is "on" and P_2 is "off," while $U=-1$ indicates the opposite.

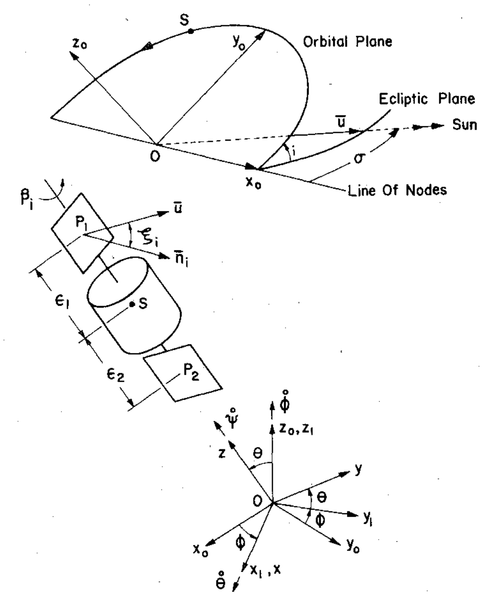

Fig. 1 Geometry of orbital and attitude motion of axisymmetric spinning satellite.

The determination of the radiation torque about the satellite center of mass is straightforward.[5] For control surfaces of high reflectivity, it is found to be

$$\bar{M} = U(2\rho p A_i \epsilon_i)\, |\cos\xi_i|\cos\xi_i(\cos\beta_i \bar{i} + \sin\beta_i \bar{j}) \tag{1}$$

where

$$\cos\xi_i = -u_x \sin\beta_i + u_y \cos\beta_i \tag{2}$$

The equations governing the satellite attitude in the presence of the radiation torque may be obtained using the Lagrangian formulation in conjunction with the principle of virtual work. The ψ degree of freedom, being cyclic, leads to the constancy of the spin rate ω_z. The equations of motion for the elevation θ and the azimuth ϕ finally take the form:

$$I_t(\ddot\theta - \dot\phi^2 \sin\theta\cos\theta) + I_s\omega_z\dot\phi\sin\theta = U(2\rho p A_i\epsilon_i)$$
$$\times\, |-u_x\sin\beta_i + u_y\cos\beta_i|\,(-u_x\sin\beta_i + u_y\cos\beta_i)\cos\beta_i \tag{3a}$$

$$I_t(\ddot\phi\sin\theta + 2\dot\phi\dot\theta\cos\theta) - I_s\omega_z\dot\theta = U(2\rho p A_i\epsilon_i)$$
$$\times\, |-u_x\sin\beta_i + u_y\cos\beta_i|\,(-u_x\sin\beta_i + u_y\cos\beta_i)\sin\beta_i \tag{3b}$$

These nonlinear coupled equations, in general, do not possess a known closed-form solution. One is, therefore, forced to resort to a numerical approach to gain some appreciation as to the system behavior.

Control Synthesis

The objective of the controller is to drive the spacecraft spin-axis from an initial attitude θ_0, ϕ_0 to the desired attitude θ_d, ϕ_d. A time-history $U=U(t)$, $\beta_i=\beta_i(t)$ accomplishing this would represent an open-loop system. On the other hand, a control law sensitive to the present attitude θ,ϕ would be self-correcting to any disturbances during the control process. To achieve a closed-loop realization for the system, the asymptotic stability criterion for the error angular momentum[7,8] is applied here.

Letting $\bar{H}$ and $\bar{H}_d$ represent the angular momentum in the present and the desired state, the error is given by

$$\bar{E} = \bar{H} - \bar{H}_d \tag{4}$$

Noting that $d\bar{H}/dt = \bar{M}$; and $d\bar{H}_d/dt = 0$ for an inertially fixed desired orientation, the condition for asymptotic decay of E^2 may be expressed as

$$dE^2/dt = 2\bar{E}\cdot\bar{M} < 0 \tag{5}$$

Substituting for $\bar{H}$ and $\bar{H}_d$ in Eq. (4) gives

$$\bar{E} = I_t(\omega_x\bar{i} + \omega_y\bar{j}) + I_s\omega_z\bar{k} - I_s\omega_z\bar{k}_d \tag{6}$$

where $\bar{k}_d$ is the unit vector along the desired spin-axis orientation.

Expressing $\bar{k}_d$ in terms of components along $\bar{i},\bar{j},\bar{k}$ in Eq. (6) and using Eq. (1), the product $\bar{E}\cdot\bar{M}$ may be formed. On substituting the kinematic relations for ω_x, ω_y, and ω_z, the resulting expression is found to be very complicated. At this stage, it is useful to assume that ω_x, $\omega_y << \omega_z$. This is equivalent to ignoring the transverse components in Eq. (6). It is apparent that the assumption remains valid except when $\bar{k}$ is very close to $\bar{k}_d$, a condition representing the termination of the control process. The criterion [Eq. (5)] subsequent to further algebraic manipulations leads to

$$dE^2/dt = U(\rho pA_i\epsilon_i I_s\omega_z)\,|-u_x\sin\beta_i$$
$$+u_y\cos\beta_i|\,[(a\,u_y - bu_x) + (au_y + bu_x)\cos 2\beta_i$$
$$-(a\,u_x - b\,u_y)\sin 2\beta_i] < 0 \qquad (7)$$

where the functions a and b are given by

$$a = \sin\theta_d\,\sin(\phi - \phi_d)$$
$$b = \sin\theta_d\,\cos\theta\,\cos(\phi - \phi_d) - \sin\theta\,\cos\theta_d \qquad (8)$$

The control law for U then follows immediately as

$$U = -\mathrm{sgn}[(au_y - bu_x) + (au_y + bu_x)\cos 2\beta_i$$
$$-(au_x - bu_y)\sin 2\beta_i] \qquad (9)$$

The choice of a control relation for the plate rotation β_i is still open. It must, however, be such that the system [Eqs. (3) and (9)] possesses $\theta = \theta_d$, $\phi = \phi_d$ as its unique equilibrium position. Singularities corresponding to $\xi_i = \pi/2$ may be avoided by selecting a time-varying β_i. Control relations of the form $\beta_i = Kt$ (K = constant) were considered and the responses indicated the policy to be effective.

For some combinations of the system parameters, however, the spin-axis came too close to the sun-line during the attitude maneuver and the controller became ineffective. This may be avoided by governing the solar torque $\bar{M}$ such that it causes the angular momentum $\bar{H}$ to approach the state $\bar{H}_d$ along the $\bar{H}$, $\bar{H}_d$ plane. The controller would then always be successful except when the sun-line makes a shallow angle with the $\bar{H}$, $\bar{H}_d$ plane. For ω_x, $\omega_y << \omega_z$, this is achieved by maintaining $\bar{n}_i$ normal to the $\bar{k}$, $\bar{k}_d$ plane which, in turn, requires β_i to be controlled as

$$\beta_i = \tan^{-1}(b/a) \qquad (10)$$

Since $\bar{H}$ has been identified with $\bar{k}$ in the derivation of Eq. (10), this is precisely the condition leading to a planar precession of the spin-axis from the attitude $\bar{k}$ to $\bar{k}_d$. Hence, it follows that during any specified attitude maneuver no switch of the active control plate would be required, which is desirable from a practical viewpoint. The control law [Eq. (9)], however, is still necessary to determine the control surface to be operated. With β_i given by Eq. (10), the control relation for U reduces to

$$U = -\mathrm{sgn}\,(a\,u_y - b\,u_x) \qquad (11)$$

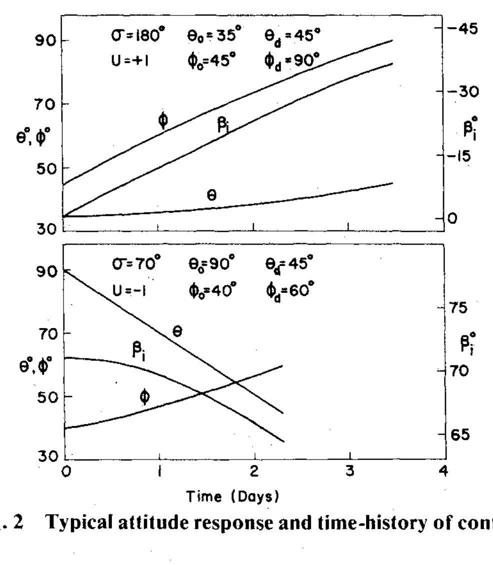

Fig. 2 Typical attitude response and time-history of controls.

It is interesting to note that situations exist where Eq. (10) leads to a constant value for the rotation β_i. For example, if a θ-control is desired with ϕ = constant, the control plate simply needs to be set at $\beta_i = \pi/2$. The same holds good when the desired spin-axis orientation is normal to the orbit plane ($\theta_d = 0$). A ϕ-control in the orbit plane ($\theta = \pi/2$) is similarly accomplished with the plate setting $\beta_i = 0$.

Controller Performance

With the present control strategy, the angle of incidence ξ_i would be constant during the control process. This leads to a constant radiation torque $\bar{M}$ (Eq. 1) causing a steady planar precession of the spin-axis at the rate $|\bar{M}|/I_s\omega_z$. The time required to drive the spacecraft spin-axis from any attitude θ, ϕ to θ_d, ϕ_d then becomes

$$t_c = (I_s\omega_z\alpha/2\rho pA_i\epsilon_i)\,(a^2 + b^2)/(a\,u_y - b\,u_x)^2 \qquad (12)$$

where α, the angle between the two orientations, is given by

$$\alpha = \cos^{-1}[\sin\theta\,\sin\theta_d\,\cos(\phi - \phi_d) + \cos\theta\,\cos\theta_d] \qquad (13)$$

The control time depends, in addition to the satellite and controller parameters, on the orbital inclination i and the apparent position of the sun indicated by the solar aspect angle σ.

As an illustration, consider a satellite in the equatorial plane ($i = 23.5$ deg) with $I_s = 10$ Kg-m^2, $I_t = 8$ kg-m^2, $\omega_z = 3$ rpm, and the controller dimensions $A_i = 0.6$ m^2, $\epsilon_i = 3$ m. With the sun location at $\sigma = 45$ deg, the time taken to align the spin-axis from the orbit normal attitude $\theta = 0$ to the in-orbit-plane orientation $\theta_d = \pi/2$, with the azimuth $\phi = 60$ deg, is obtained as $\simeq 4$ days. This appears encouraging, as control times of this order may be acceptable for long-life multiple-mission spacecraft.

The elevation and azimuth response for the example satellite, obtained by integrating Eqs. (3) in conjunction with the control relations [Eqs. (10) and (11)], is shown in Fig. 2 for two different attitude maneuvers. For any other values of satellite inertia, spin-rate, and controller dimensions, the response would remain identical except for a change of time scale proportional to the quantity $(I_s\omega_z/A_i\epsilon_i)$. The figure also indicates the plate polarity U and the rotation β_i as functions of time. As mentioned earlier, no switch of the control U is necessary for transition between any two specified orientations. The residual spin-axis motion in the neighborhood of the desired attitude was also found to be quite negligible. This, of course, could have been anticipated in view of the rather slow rate of forced precession.

In summary, the feasibility of using solar radiation pressure to provide any inertially fixed orientation to the satellite spin-axis is demonstrated. The controller imparts versatility to the spacecraft, enabling it to undertake diverse missions. As no mass expulsion schemes or active devices requiring large power consumption are involved, the system is essentially semipassive. This promises an increased satellite life-span.

References

[1]Ule, L. A., "Orientation of Spinning Satellites by Radiation Pressure," *AIAA Journal*, Vol. 1, July 1963, pp. 1575-1578.

[2]Peterson, C. A., "Use of Thermal Radiation Effects in Spacecraft Attitude Control," Center for Space Research, MIT, Cambridge, Mass., CSRR-66-3, 1966.

[3]Colombo, G., "Passive Stabilization of a Sunblazer Probe by Means of Radiation Pressure Torque," Center for Space Research, MIT, Cambridge, Mass., CSR TR-66-5, 1966.

[4]Falcovitz, J., "Attitude Control of a Spinning Sun-orbiting Spacecraft by Means of a Grated Solar Sail," Center for Space Research, MIT, Cambridge, Mass., CSR TR-66-17, 1966.

[5]Modi, V. J. and Pande, K. C., "Solar Pressure Control of a Dual-Spin Satellite," *Journal of Spacecraft and Rockets*, Vol. 10, June 1973, pp. 355-361.

[6]Pande, K. C., Davies, M. S., and Modi, V. J., "Time-Optimal Pitch Control of Satellites Using Solar Radiation Pressure," *Journal of Spacecraft and Rockets*, Vol. 11, Aug. 1974, pp. 601-603.
[7]Ergin, E. I. and Wheeler, P. C., "Magnetic Attitude Control of a Spinning Satellite," *Journal of Spacecraft and Rockets*, Vol. 2, Nov. – Dec. 1965, pp. 846-850.
[8]Shigehara, M., "Geomagnetic Attitude Control of an Axisymmetric Spinning Satellite," *Journal of Spacecraft and Rockets*, Vol. 9, June 1972, pp. 391-398.

Technical Comments

Spacecraft Radio-Occultation Technique for the Study of Planetary Atmospheres

Von R. Eshleman*
Stanford University, Stanford, Calif.

IN the 12th von Karman lecture,[1] Schurmeier presents a beautifully descriptive and inspiring account of this nation's program of planetary exploration. I applaud the total result, but wish to expand upon one point. On page 392 he notes that "A striking new technique...is radio oncultation measurements of atmospheric density...conceived shortly before the 1964 launch (of Mariner IV) by D.L. Cain of JPL...". He then goes on to discuss other aspects of the experiment and trajectory for the 1965 radio occultation measurements of the atmosphere and ionosphere of Mars. I believe that more should be said about this first demonstration involving radio occultation of planetary spacecraft, and in particular about the origins of the concept.

A sequence of numerous meeting papers and discussions, letters, proposals, and memos on the subject of "bistatic radar astronomy" was begun by Stanford colleagues and myself in 1960. This phrase was defined in terms of several space radioscience experiments made possible by the use of propagation between two radio terminals, one on Earth and one on a spacecraft near a planetary target One such experiment is radio occultation for the study of planetary atmospheres and ionospheres. (This differs from earlier optical and radio star occultation experiments in that it is based upon the use of a point source of coherent radiation, so that signal frequency as well as intensity is an observable.) For example,I discussed this field at the June 1960 meeting at JPL on electromagnetic studies of the moon and planets, and introduced the subject at the Space Science Summer Study of the National Academy of Sciences held at the State University of Iowa in June-August 1962.[2] Also in 1962, Gunnar Fjeldbo began working on his Stanford PhD dissertation, which was to include a very detailed analysis of the radio occultation concept emphasizing the study of planetary ionospheres and surfaces.[3] That same year we initiated what became a continuing discussion with JPL and NASA representatives on the subject of spacecraft trajectories designed for occultation experiments, and work was started at Stanford on dual-frequency equipment for such studies.

It appears that the statement by the author which is quoted above must refer to the implementation of the experiment for Mariner IV and not to the basic radio occultation concept. I agree that Cain and his colleagues at JPL made original contributions which were of fundamental importance for the conduct of this initial experiment. They conceived and developed the technique that used the JPL coherent counted doppler system to analyze the planetary atmosphere during an occultation. This technique enabled a direct determination of the refractivity of the neutral atmosphere and it did not require additional flight equipment, thus making an occultation experiment feasible on Mariner IV. However, with reference to radio occultation as a method for the study of planetary atmospheres and ionospheres, I would distinguish conception of the method from conception of a technique for implementing the method, with the former taking place several years earlier.

Received April 23, 1976; revision received Aug. 17, 1976
Index categories; Atmospheric, Space, and Oceanographic Sciences; Lunar and Planetary Spacecraft Systems, Unmanned; Spacecraft Mission Studies and Economics.

*Director, Radioscience Laboratory; Co-director, Center for Radar Astronomy; Professor, Department of Electrical Engineering.

References

[1]Schurmeier, H.M., "Planetary Exploration: Earth's New Horizon," *Journal of Spacecraft and Rockets*, Vol. 12, July 1975, pp. 385-405.
[2]"A Review of Space Research," Publication 1079, National Academy of Sciences-National Research Council, Washington, D.C., 1962, pp. 6-1 to 6-9.
[3]Fjeldbo, G., "Bistatic Radar Methods for Studying Planetary Ionspheres and Surfaces," Stanford Electronics Laboratories, Stanford, Calif., SU-64-025, April 1964.

Errata

Estimation of Satellite Lifetime from Orbital Failure Experience

D. A. Conrad
The Aerospace Corporation, El Segundo, Calif.

[JSR, **13**, 75-81 (1976)]

FIGURE 5 at the bottom of column one on page 79 was improperly captioned. The correct caption is:

Fig. 5 Probability of launch for random truncation and random failure rate factors.

Received June 28, 1976.
Index category: Reliability, Quality Control, and Maintainability.

Announcement: 1976 Author and Subject Index

The indexes of the four AIAA archive journals (*AIAA Journal, Journal of Spacecraft and Rockets, Journal of Aircraft,* and *Journal of Hydronautics*) will be combined and mailed separately early in 1977. In addition, papers appearing in volumes of the *Progress in Astronautics and Aeronautics* book series published in 1976 will be included. Librarians will receive one copy of the index for each subscription which they have. Any AIAA member who subscribes to one or more Journals will receive one index. Additional copies may be purchased by anyone, at $10 per copy, from the Circulation Department, AIAA, Room 730, 1290 Avenue of the Americas, New York, New York 10019. **Remittance must accompany the order.**

Ruth F. Bryans
Director, Scientific Publications